Lecture 24

Recap:

$$\Pr[E \mid F] = \frac{\Pr[E \cap F]}{\Pr[F]}$$

$\boxed{\text{Example}}$

- 0.1% of users are ND students
- 5% of people who watched Rudy are ND students
- 1% of all users watched Rudy

$$\left.\Pr[\text{Rudy} \mid ND] = \frac{\Pr[ND \mid \text{Rudy}] \cdot \Pr[\text{Rudy}]}{\Pr[ND]}\right\} \text{Bayes Theorem}$$

Random Variable, Expected Value, and Gamble

- If a variable X is random, its value is determined with some probabilities.

$$E[X] = (1) \times P[X=1] + 2 \times P[X=2] + \dots + 5 \times P[X=5] + 6 \times P[X=6]$$

value of a die

So expected value: $E[X] = 1(\frac{1}{6}) + 2(\frac{1}{6}) + \dots + 5(\frac{1}{6}) + 6(\frac{1}{6})$

$E[X] = 3.5$

Linearity

$$E[X+Y] = E[X] + E[Y]$$
$$E[cX] = cE[X]$$

- Example

- X = # of different birthdays in the room (97 people in room)
- Assuming uniform distribution, what is $E[X]$

$$E[X] = (1)\cdot P[X=1] + 2\cdot P[X=2] + \dots + 365\cdot P[X=365]$$

or

Define Indicator variable $I_y = \begin{cases} 1 & \text{someone was born on day } y \\ 0 & \text{no one was born on day } y \end{cases}$

$$X = I_1 + I_2 + \dots + I_{364} + I_{365}$$

$$E[X] = E[I_1 + \dots + I_{365}] = E[I_1] + E[I_2] + \dots + E[I_{365}]$$

arbitrary day 16 $\rightarrow$ $E[I_{16}] = 0 \times P[I_{16}=0] + 1 \times P[I_{16}=1]$

$$P[I_{16}=1] = 1 - P[I_{16}=0]$$
$$= \underbrace{1 - \left(\tfrac{364}{365}\right)^{97}}$$

This is the value for $P[I_1=1] = P[I_2=1] = \dots = P[I_{365}=1]$

- So $E[X] = 365\left[1 - \left(\frac{364}{365}\right)^{97}\right]$

- Indicator variable and expected values will likely be on final exam.

- All expected values of casino games are negative.

- Algorithm Taeho uses to make money in Vegas.

1. Sit on roulette Table (minimum bet is $1)
2. Bet $1 on red or black
   - ↳ If you lose, double the bet and repeat step 2
   - ↳ If I win, reset the bet to $1 and repeat step 2

- Casinos know this, thats why they put high limits.